X056/301

NATIONAL
QUALIFICATIONS
2000

THURSDAY, 25 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 There are three Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**
either Section B (Mathematics 3)
or Section C (Statistics).

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives and integrals:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

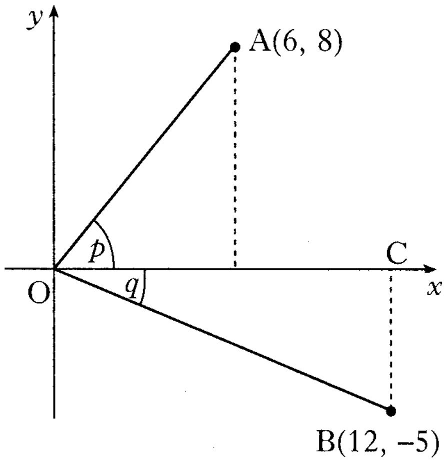
SECTION A (Mathematics 1 and 2)

Marks

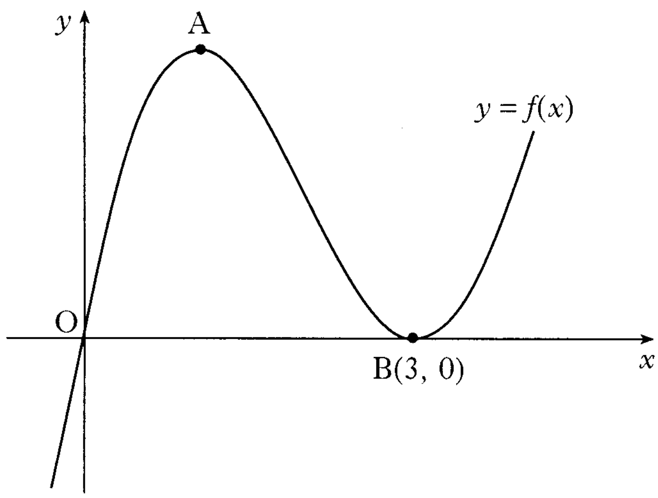
ALL candidates should attempt this Section.

A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. **4**



A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(a) Find the coordinates of the turning point at A. **4**

(b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

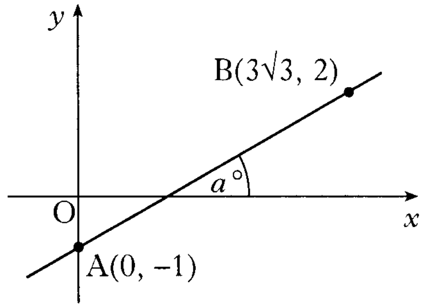
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(c) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

[Turn over

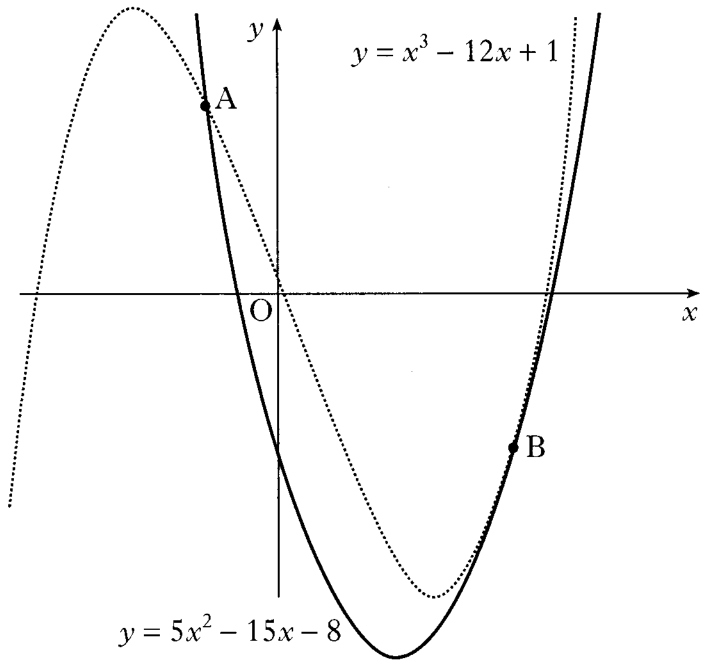
Marks

A3. Find the size of the angle $a°$ that the line joining the points A(0, −1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. **3**



A4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is (−1, 12) and B is (3, −8).

Find the area enclosed between the two curves. **5**

Marks

A5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

[*END OF SECTION A*]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Page six*

OR Section C (Statistics) on *Pages seven and eight*

[Turn over

SECTION B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

B7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

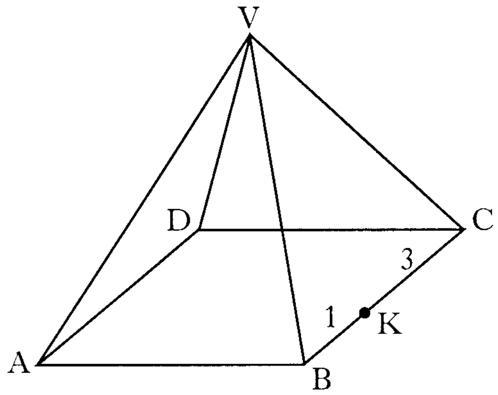
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form. **3**



B8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

B9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

B10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

[*END OF SECTION B*]

X056/302

NATIONAL
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2000

THURSDAY, 25 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

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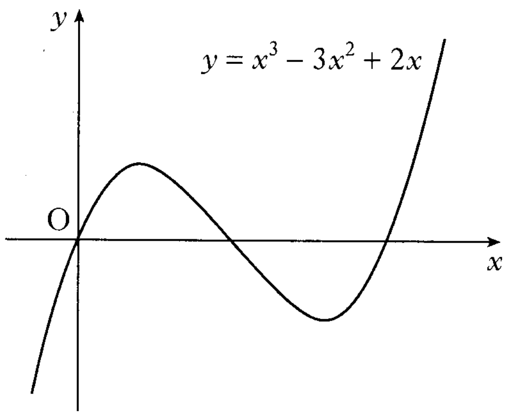
SECTION A (Mathematics 1 and 2)

Marks

ALL candidates should attempt this Section.

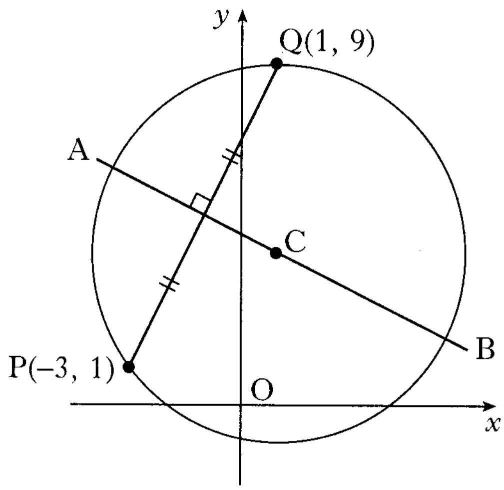
A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

A2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). **4**



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**

A3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

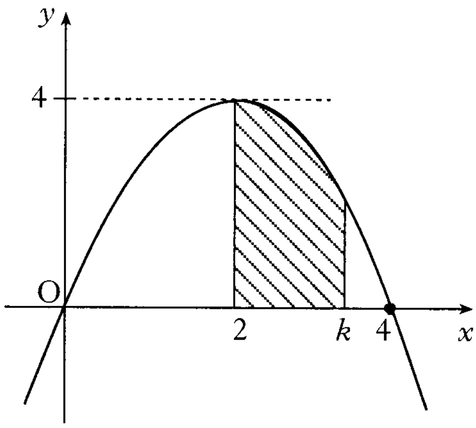
(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \frac{3}{3 - x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Marks

A4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(a) Find the equation of the parabola. **2**

(b) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

A5. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. **5**

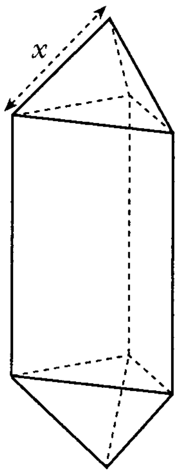
A6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



[END OF SECTION A]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Pages five and six*

OR Section C (Statistics) on *Pages seven and eight*

SECTION B (Mathematics 3)

Marks

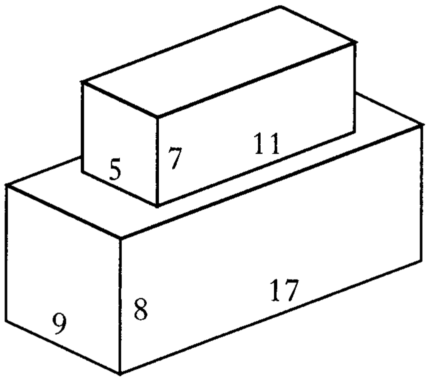
ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

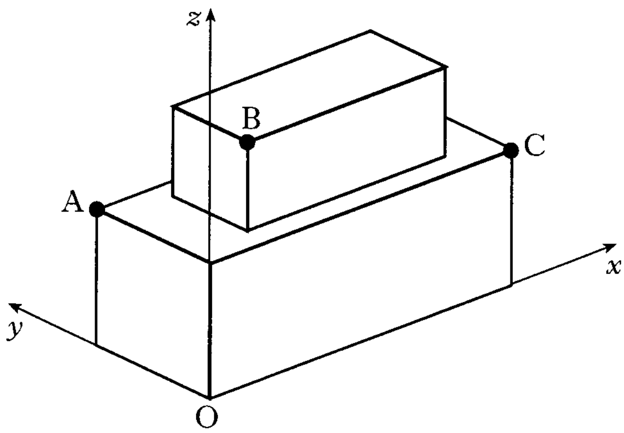
B7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

B8. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

B9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

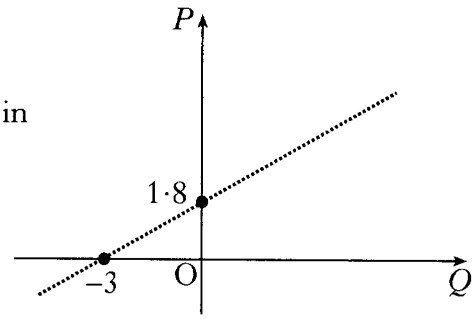
[Turn over

Marks

B10. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

B11. The results of an experiment give rise to the graph shown.

(*a*) Write down the equation of the line in terms of P and Q. **2**



It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

[*END OF SECTION B*]